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Path Tracking Control of a Manipulator with Passive Joints

Hirohiko ARAI*, Kazuo TANIE*, and Susumu TACHI**

* Mechanical Engineering Laboratory, AIST, MITI, 1-2 Namiki, Tsukuba, Ibaraki 305 Japan
** RCAST, The University of Tokyo, 4-6-1 Komaba, Meguro-ku, Tokyo 153 Japan

Abstract

A method is proposed of path tracking control of a manipulator with passive joints, i.e. having no actuators. A desired path is geometrically specified in operational space. The position of the manipulator is controlled to follow the desired path. In this method, a *path coordinate system* based on the desired path is defined in operational space. The path coordinates consist of a component parallel to the desired path and components normal to the desired path. The equation of motion of the manipulator is described in terms of the path coordinates. The acceleration of the components normal to the desired path is controlled according to feedback based on tracking error by using the dynamic coupling among the components. This in turn keeps the manipulator on the desired path. Results of path tracking experiments using a two-degree-of-freedom manipulator with a passive joint are presented.

1. Introduction

The number of degrees of freedom of a conventional manipulator is equal to the number of joint actuators. Since the mass of the actuator of a serial-type manipulator is a load for the next actuator, the size of the actuator should increase rapidly from the wrist joint to the base joint. As a result, the base joint must be equipped with a huge actuator compared to the load of the manipulator. In order to decrease the weight, cost, and energy consumption of a manipulator, various methods have been proposed for controlling a manipulator which has more degrees of freedom than actuators [1]. However, these methods require special mechanisms in addition to the basic links and joints. In this paper, a method is presented for controlling a manipulator which has more joints than actuators without using additional mechanisms.

The dynamics of a manipulator have non-linear and coupling characteristics. When each joint is controlled by a local linear feedback loop, these factors result in disturbances. The elimination of such dynamic disturbances has been one of the major problems in manipulator control [2]-[4]. However, the effects of these disturbances are available to drive a joint which in itself does not have an actuator. Such dynamic characteristics are actively used in human handling tasks. For example, when a heavy load is handled, all the muscles of the human arm are not necessarily used. Some joints, e.g. wrist joints, are kept free and the inertia of the load is utilized effectively. Such a *"dynamic skill"* will also be significant for robot control. Some robot control schemes which utilize dynamic coupling effects have previously been proposed [5],[6].

As a means of controlling a manipulator which has more joints than actuators without using additional mechanisms, we have proposed a method for controlling passive joints by using dynamic coupling [7]. We also developed an algorithm for point-to-point control of the manipulator and applied it to a two-degree-of-freedom manipulator [8]. In this method, a manipulator is composed of two types of joints, active and passive joints. Each active joint consists of an actuator and a position sensor (e.g. an encoder). Each passive joint consists of a holding brake and a position sensor. When the brakes of the passive joints are engaged, the active joints can be controlled without affecting the state of the passive joints. When the brakes are released, the passive joints can rotate freely. The motion of the active joints generates acceleration of the passive joints via the coupling characteristics of manipulator dynamics. The passive joints can be controlled indirectly in this manner. The total position of the manipulator is controlled by combining these two control modes. Jain et.al. independently proposed a similar technique to control a manipulator with passive hinges. They also developed an efficient algorithm using spatial operator algebra [9].

When some of the joint actuators of a manipulator are exchanged for holding brakes with this method, we can build a lightweight, energy-saving, low-cost manipulator. We can take advantage of these merits by applying the method to simple assembly robots, control of redundant manipulators, etc. Space applications (e.g. space manipulators, expansion of space structures) may be feasible. This method can also be applied to failure recovery control of a manipulator [10].

Control with the passive joints released is an essential part of this method. In Refs. [7], [8], we controlled the manipulator in joint space. In this approach, a desired trajectory is assigned to the passive joints and the motion and torque of the active joints is calculated to realize the desired motion of the passive joints. The motion of the active joints is determined by the desired trajectory of the passive joints and the dynamic coupling among the joints. Consequently, the motion of the tip of the manipulator cannot be prescribed. However, the position of the tip in operational space, e.g. Cartesian space, is usually important for practical manipulator tasks. It is necessary to control the path along which the tip moves if the manipulator is to avoid collision with an obstacle.

We recently proposed a method for controlling the position of a manipulator with passive joints, not in joint space, but in operational space [11]. In this method, the equation of motion is represented in terms of operational coordinates. The operational coordinates are separated into controlled components and compensating components. The desired acceleration can be generated at the controlled components, which are equal in number to the active joints, by using dynamic coupling among the components.

In this paper, this method is extended to path tracking control of a manipulator with passive joints. A desired path is geometrically specified in operational space. The tip position of the manipulator is controlled to follow the desired path. In this method, *path coordinates* are defined as a kind of operational coordinate system based on the desired path. The path coordinates consist of a component parallel to the desired path and components normal to the desired path. The equation of motion of the manipulator is described in terms of the path coordinates. The acceleration of the components normal to the desired path is controlled by using the dynamic coupling among components. This in turn keeps the manipulator on the desired path. The effectiveness of the proposed method is demonstrated by experiments using a two-degree-of-freedom manipulator with a passive joint.

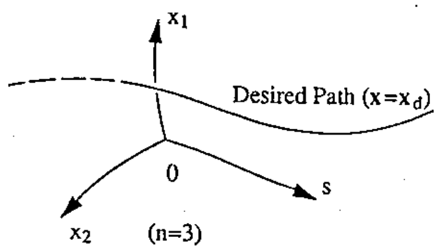


Fig.1 Path Coordinate System

2. Path Coordinates

A mathematical description of the desired path of a manipulator is considered in this section. The desired path is geometrically specified as a continuous curve in operational space. It is not associated with a time variable. In minimum-time trajectory planning problems, this type of path is often parametrized by a path parameter [12]-[14]. The position of a point on the path is represented as a vector function of a scalar parameter. When operational space is n-dimensional, a point $q \in R^n$ on the path is represented as

$$q = q(s), s_0 \le s \le s_f \tag{1}$$

where s is the path parameter. $q(s_0)$ is the start point of the path and $q(s_f)$ is the end point. s can be considered as a distance along the path. Since s is a scalar, this method can represent a point only on the path itself.

Real-time path tracking control is considered here. If the manipulator deviates from the path due to disturbances, feedback control should force the manipulator to return to the path. Therefore, points which are not on the path as well as points on the path should be represented. Furthermore, the tracking error should be measured. We propose the concept of *path coordinates* as an extension of the path parameter. A curvilinear coordinate frame is defined in operational space. The coordinates are composed of a component s along the path and components $x_1, \ldots, x_{n-1}$ normal to s. These coordinates are called path coordinates (Fig.1). A point $p \in R^n$ represented in terms of the path coordinates is

$$p = [x_1, \ldots, x_{n-1}, s]^T = [x^T \ \ s]^T \tag{2}$$

The desired path is represented as

$$x = x_d \text{ (constant)}, s_0 \le s \le s_f \tag{3}$$

in terms of the path coordinates. The desired path is also represented as

$$q = q([x_d^T \ \ s]^T), s_0 \le s \le s_f \tag{4}$$

in terms of the operational coordinates. Eq.(4) represents all points in operational space.

$$q = q(p) \tag{5}$$

Eq.(5) represents the coordinate transformation from the path coordinate space to operational space.

The motion on the path is of the s component only and has one degree of freedom. It corresponds to motion on the straight line, $x = x_d$, in the path coordinate space. It means that x components of the path coordinates always remain constant values, x_d, if the manipulator is on the path. $x - x_d$ represents the deviation of the point, $p = [x^T \ \ s]^T$, from the desired path, $x = x_d$.

3. Equation of Motion in terms of Path Coordinates

Both the operational space and the path coordinate space are assumed to be n-dimensional. It is assumed that the manipulator also has n degrees of freedom and it consists of n-1 active joints and one passive joint. The equation of motion of the manipulator in joint space can be written as follows

$$M(\theta)\ddot{\theta}+b(\theta,\dot{\theta})=u \quad (6)$$

where

$b(\theta,\dot{\theta})=h(\theta,\dot{\theta})+\Gamma\dot{\theta}+g(\theta)$
$\theta\in R^{n}$: joint angle vector
$u\in R^{n}$: joint torque vector
$g(\theta)\in R^{n}$: gravity torque vector
$h(\theta,\dot{\theta})\in R^{n}$: Coriolis and centrifugal torque vector
$M(\theta)\in R^{n\times n}$: inertia matrix
$\Gamma\in R^{n\times n}$: viscous friction matrix

The joint torque vector u is composed of the active joint torque, $\tau\in R^{n-1}$, and the passive joint torque (= 0).

$$u=[\tau^{T}\ \ 0]^{T} \quad (7)$$

The equation of motion of the manipulator is rewritten in terms of path coordinates $p\in R^{n}$. We assume that the path coordinates and the joint coordinates are related as follows

$$\dot{p}=J\dot{\theta} \quad (8)$$

where $J\in R^{n\times n}$ is a Jacobian matrix. When eq.(8) is differentiated with respect to time, we obtain

$$\ddot{p}=J\ddot{\theta}+\dot{J}\dot{\theta} \quad (9)$$

Note that the manipulator has n degrees of freedom and is non-redundant. If J is non-singular

$$\ddot{\theta}=J^{-1}(\ddot{p}-\dot{J}\dot{\theta}) \quad (10)$$

Here, M, b, and $H\equiv J^{-1}$ are partitioned as follows

$$M=\begin{bmatrix}M_{l}\\M_{2}\end{bmatrix}\begin{matrix}n\text{-}1\\1\end{matrix} \qquad b=\begin{bmatrix}b_{l}\\b_{2}\end{bmatrix}\begin{matrix}n\text{-}1\\1\end{matrix} \quad (11)$$

$$H=\underset{\;\;n-1\;\;\;1}{[H_{l}\ \ H_{2}]}\ n$$

When eq.(7), (10), and (11) are substituted into eq.(6) we obtain

$$M_{l}H_{l}\ddot{x}+M_{l}H_{2}\ddot{s}-M_{l}H\dot{J}\dot{\theta}+b_{l}=\tau \quad (12a)$$

$$M_{2}H_{l}\ddot{x}+M_{2}H_{2}\ddot{s}-M_{2}H\dot{J}\dot{\theta}+b_{2}=0 \quad (12b)$$

The equation of motion is represented in terms of the path coordinates. It is divided into eq.(12a), which is related to the torque of the active joints, and eq.(12b), which is related to the torque of the passive joint.

4. Path Tracking Control

In this section, a control method is proposed in which the manipulator described in Section 3 tracks the desired path defined in Section 2. As an initial condition, it is assumed that the manipulator is moving with sufficient initial velocity in the direction of the path when the path tracking control begins. This can be achieved, for example, by accelerating the active joints while the passive joint is fixed before the path tracking control commences with the passive joint released.

It is shown that the desired acceleration can be generated arbitrarily at components which are equal in number to the active joints. The number of active joints of the present manipulator is n-1. The number of components of the path coordinates is n, and the number of components normal to the path is n-1. We designed a control system which gives priority to the components x which are normal to the path. It generates acceleration of the x components in order to keep the manipulator on the path.

From eq.(3), x is constant when the manipulator moves along the desired path. Thus $\dot{x}=0$ and $\ddot{x}=0$ on the path. It is assumed that $M_2H_2\neq 0$. Since $H\dot{J}=-\dot{H}J$, acceleration along the path is

$$\ddot{s}=-(M_{2}H_{2})^{-1}(M_{2}\dot{H}_{2}\dot{s}+b_{2}) \quad (13)$$

from eq.(12b). The trajectory of s is determined if the initial values of s and $\dot{s}$ are given. Thus, the time trajectory of the manipulator is obtained. From eq.(12a), the torque of the active joints is

$$\tau=\{M_{l}-M_{l}H_{2}(M_{2}H_{2})^{-1}M_{2}\}\dot{H}_{2}\dot{s}+b_{l}-M_{l}H_{2}(M_{2}H_{2})^{-1}b_{2} \quad (14)$$

In the case of open loop control, the manipulator may deviate from the desired path due to disturbances or modeling error. We designed a closed loop control system to suppress this tracking error. In order to keep the manipulator on the desired path, the x components normal to the path should remain constant at x_d. The acceleration $\ddot{x}_d$ is generated to adjust x to x_d. The following PID control is applied

$$\ddot{x}=-K_{v}\dot{x}+K_{p}(x_{d}-x)+K_{i}\int(x_{d}-x)dt \quad (15)$$

where K_v, K_p, and $K_i\in R^{(n-1)\times(n-1)}$ are the diagonal gain matrices. x and $\dot{x}$ are the measured values of the position and velocity of the x components. The component s along the path is controlled so as to realize the appropriate acceleration of x components in eq.(15).

When the measured values of the joint angle and velocity are substituted into θ and $\dot{\theta}$ of eq.(12a) and (12b), each component of M, H, J, and b is determined. If the acceleration of the x components in eq.(15) are assigned to $\ddot{x}$, eq.(12b) is a linear equation with respect to $\ddot{s}$. If M_2H_2 is not equal to zero, eq.(12b) can be solved uniquely as

$$\ddot{s}=(M_{2}H_{2})^{-1}(-M_{2}H_{l}\ddot{x}+M_{2}H\dot{J}\dot{\theta}-b_{2}) \quad (16)$$

When eq.(16) is substituted into eq.(12a), the torque τ to realize the acceleration in eq.(15) can be determined.

$$\tau = \{M_1 - M_1H_2(M_2H_2)^{-1}M_2\}\ (H_1\ddot{x} - H\dot{J}\dot{\theta}) + b_1 - M_1H_2(M_2H_2)^{-1}b_2 \quad (17)$$

When we apply this torque τ of eq.(17) to the active joints, we will obtain the x components acceleration of eq.(15). When the deviation of the manipulator from the desired path is $e = x - x_d$, from eq.(15)

$$\ddot{e} + K_v\dot{e} + K_p e + K_i\int e dt = 0 \quad (18)$$

The deviation e is guaranteed to converge to zero if K_v, K_p, and K_i are chosen such that all the poles of the system (18) are located in the left-half plane. As a result, the manipulator tracks the desired path.

In this method, the s component is accelerated / decelerated in order to realize the desired value of the x components. Therefore the s component cannot be controlled directly. The time period necessary to travel along the path depends on the configuration of the path and the initial velocity along the path. The initial value of component s is s_0 and the final value is s_f. When initial values s_0 and $\dot{s}_0$ are used in eq.(13)

$$\ddot{s} = -(M_2H_2)^{-1}(M_2\dot{H}_2\dot{s} + b_2)$$

and s and $\dot{s}$ are calculated by numerical integration for $s_0 \le s \le s_f$, the final value $\dot{s}_f$ and the time period to travel along the path is obtained.

If $|\dot{s}_0|$ is too small, the direction of $\dot{s}$ may be inverted before reaching the final point and the manipulator cannot complete the path tracking. Therefore it is desirable to estimate the minimum $|\dot{s}_0|$ necessary to reach the final point. M_2 and H_2 in eq.(13) can be represented as functions of s only. On the path, $\dot{\theta} = H_2\dot{s}$. When the friction of the passive joint is negligibly small and b_2 is composed of Coriolis, centrifugal, and gravitational forces, eq.(13) can be rewritten as

$$\ddot{s} = f(s)\,\dot{s}^2 + g(s) \quad (19)$$

where $f(s)$ and $g(s)$ are functions of s and do not depend on initial velocity $\dot{s}_0$. Note that

$$\ddot{s} = \frac{ds}{dt}\cdot\frac{d\dot{s}}{ds} = \frac{1}{2}\frac{d(\dot{s})^2}{ds} \quad (20)$$

Eq.(19) is a linear differential equation with respect to $\dot{s}^2$. The solution of eq.(19) is

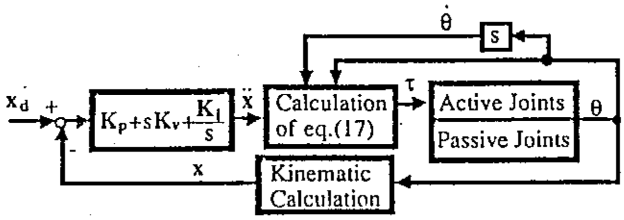


Fig.2 Path Tracking Control System

$$\dot{s}^2(s) = \exp\left(\int_{s_0}^{s} 2f(\zeta)d\zeta\right)\cdot\left[\dot{s}_0^2 + \int_{s_0}^{s} 2g(\zeta)\exp\left(-\int_{s_0}^{\zeta} 2f(\varepsilon)d\varepsilon\right)d\zeta\right] \quad (21)$$

Since $\exp\left(\int_{s_0}^{s} 2f(\zeta)d\zeta\right) > 0$, the condition for $\dot{s}^2(s) > 0$ is

$$\dot{s}_0^2 > -\int_{s_0}^{s} 2g(\zeta)\exp\left(-\int_{s_0}^{\zeta} 2f(\varepsilon)d\varepsilon\right)d\zeta \quad (22)$$

The right side of eq.(22) is calculated by numerical integration. If $\dot{s}_0$ is chosen so that $\dot{s}_0^2$ is larger than the maximum of the integration value, the manipulator can reach the final point. It is clear from eq.(22) that if $g(s) \ge 0$ for $s_0 \le s \le s_f$, the manipulator can reach the final point for any $\dot{s}_0 > 0$.

The initial velocity necessary to reach the desired velocity at the final point or at a point halfway along the path can be calculated if the initial velocity is in excess of the minimum velocity. When eq.(13) is integrated backward for $s_0 \le s \le s_d$ with s and $\dot{s}$ given final values s_d and $\dot{s}_d$, the initial velocity $\dot{s}_0$ to realize the desired velocity $\dot{s}_d$ is derived.

During real-time control,

$$\lambda = \frac{s - s_0}{s_f - s_0} \quad (23)$$

and $0 < \lambda < 1$ on the path. The value of λ is monitored while the manipulator is tracking the path. The control terminates when $\lambda > 1$.

Fig.2 shows the path tracking control system.

5. Control of a Manipulator with Multiple Passive Joints

A manipulator which is composed of n-1 active joints and one passive joint is considered in Sections 3 and 4. However, the proposed control method is easily extended to a manipulator with multiple passive joints. In n-dimensional operational space, it is necessary to control n-1 components to keep the manipulator on the desired path. Therefore, n-1 active joints are necessary. The manipulator is assumed to have $m > 1$ passive joints. The equation of motion is

$$M(\theta)\ddot{\theta} + b(\theta, \dot{\theta}) = u \quad (24)$$

$$u = [\tau^T \ \ 0]^T \quad (25)$$

where $\theta \in R^{n+m-1}$, $M(\theta) \in R^{(n+m-1)\times(n+m-1)}$, $b(\theta, \dot{\theta}) \in R^{n+m-1}$, $u \in R^{n+m-1}$, and $\tau \in R^{n-1}$.

Here, m-1 auxiliary components, $z_1, \ldots, z_{m-1}$, are added to the path coordinates p.

$$p = [x_1, \ldots, x_{n-1}, s, z_1, \ldots, z_{m-1}] = [x^T, y^T]^T \quad (26)$$

where $x \in R^{n-1}$ and $y \in R^m$. The choice of auxiliary components is arbitrary except that the Jacobian matrix J

$\in \mathbb{R}^{(n+m-1)\times(n+m-1)}$ should be non-singular. The equation of motion is partitioned as

$$M_1H_1\ddot{x} + M_1H_2\ddot{y} - M_1H\dot{J}\dot{\theta} + b_1 = \tau \qquad (27a)$$

$$M_2H_1\ddot{x} + M_2H_2\ddot{y} - M_2H\dot{J}\dot{\theta} + b_2 = 0 \qquad (27b)$$

If matrix $M_2H_2 \in \mathbb{R}^{m\times m}$ is non-singular,

$$\ddot{y} = (M_2H_2)^{-1}(-M_2H_1\ddot{x} + M_2H\dot{J}\dot{\theta} - b_2) \qquad (28)$$

The control law is the combination of PID feedback eq.(15) and the torque computation

$$\tau = \{M_1 - M_1H_2(M_2H_2)^{-1}M_2\}(H_1\ddot{x} - H\dot{J}\dot{\theta}) + b_1 - M_1H_2(M_2H_2)^{-1}b_2 \qquad (29)$$

It has almost the same form as eq.(17).

6. Experiments

We conducted experiments testing this control method with a two-degree-of-freedom horizontally-articulated manipulator. Fig.3 shows the manipulator. The first axis (θ_1) is an active joint and the second axis (θ_2) is a passive joint. The active joint is driven by a DC servo motor with a harmonic-drive gear. The brake of the passive joint is electromagnetic.

The desired path (a) is given as a straight line. In the case of (a), the path coordinate space is Cartesian space.

$$p = [x, y]^T$$
$$x = L\cos\theta_1 + L\cos(\theta_1+\theta_2)$$
$$y = L\sin\theta_1 + L\sin(\theta_1+\theta_2)$$

The desired path (a) is represented as $x_d = 0.4$(m). The desired path (b) is given as a circular arc around the first axis. In the case of (b), the path coordinate space is a polar coordinate space whose origin is at the first joint.

$$p = [r, \phi]^T$$
$$r = 2L\cos\frac{\theta_2}{2},\ \phi = \theta_1 + \frac{\theta_2}{2}$$

The desired path is $r_d = 0.4$(m). Fig.4 illustrates the results of the path tracking control experiments. The initial condition is $y_0 = 0$m, $\dot{y}_0 = 0.7$m/s in both cases. The results (solid lines) follow the desired paths (dotted lines). Trackings to the different paths are achieved from the same initial condition. The maximum deviation is 2.9×10^{-4}m in (a), and 7.2×10^{-4}m in (b). Fig.5 shows the response when the initial position is not on the path. The deviations converge to zero by means of the feedback control. Fig.6 shows tracking of a desired path which includes straight and circular path segments. In this way, a complicated desired path can be composed of several simple path segments.

7. Conclusions

A method of path tracking control of a manipulator with passive joints is proposed. A *path coordinate system* based on the desired path is defined. The equation of motion of the manipulator is described in terms of the path coordinates. The acceleration of the components normal to the desired path is controlled by using the dynamic coupling among components. This in turn keeps the manipulator on the desired path. The effectiveness of the method was verified by experiments using a two-degree-of-freedom manipulator with a passive joint. The experiments showed that the path of the manipulator can be controlled precisely by use of the proposed method. Since this method uses closed-loop control, precise tracking is possible in the presence of disturbances. The path of the manipulator can be prescribed with this method and it may be combined with obstacle avoidance or other path-planning algorithms.

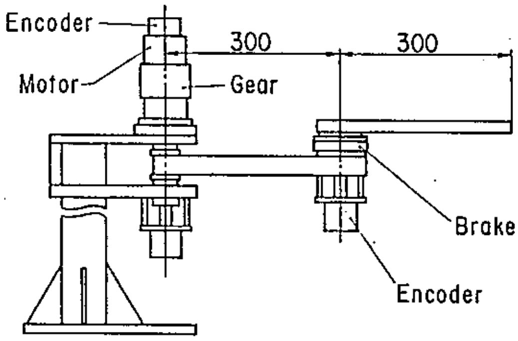
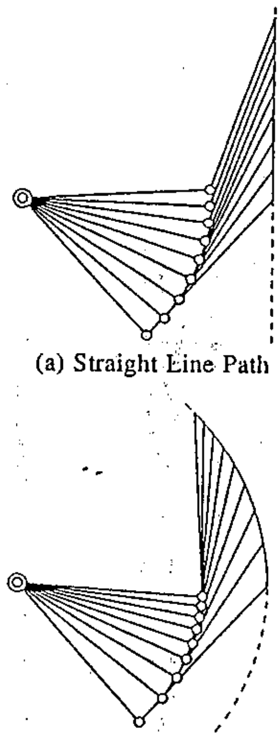


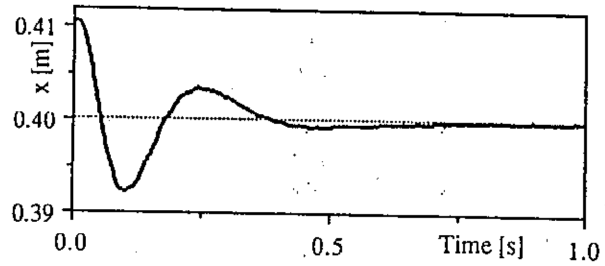
Fig.3 Two-Degree-of-Freedom Manipulator



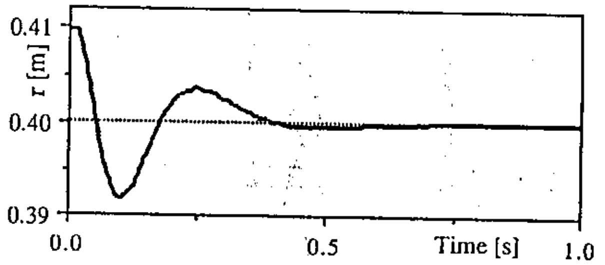
(a) Straight Line Path

(b) Circular Arc Path

Fig.4 Path Tracking Motion



(a) Straight Line Path



(b) Circular Arc Path

Fig.5 Suppression of Tracking Error

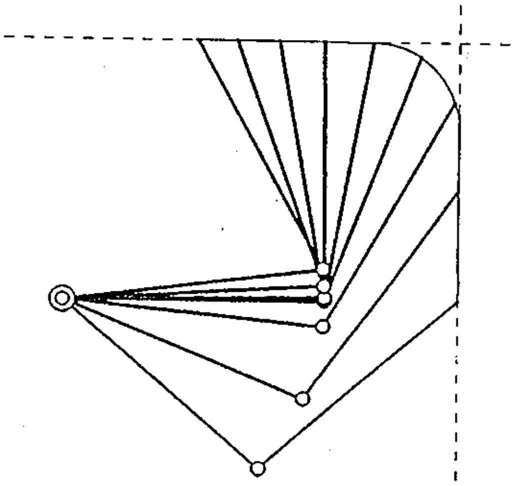


Fig.6 Tracking of a Composite Path

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