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Paper

Basic considerations of the degrees of freedom of multi-legged locomotion machines

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Abstract—All multi-legged locomotion machines that do not need any dynamic balance control can be classified functionally into several levels. We define the minimum walking functions of multi-legged locomotion machines as follows: (i) two-dimensional walking; (ii) keeping the body horizontal on irregular terrains; (iii) keeping the absolute height of main body constant.

Our main interest is in how many active degrees of freedom are necessary and sufficient to realize the above functions. Although consideration of the degrees of freedom seems to be fundamental in developing multi-legged locomotion machines, this problem has not yet been studied. The active degrees of freedom are examined in this paper using a four-legged machine which offers the minimum number of legs necessary to maintain static stability. It is shown that six active degrees of freedom are necessary and sufficient to realize the above functions.

1. INTRODUCTION

In a manipulator, working functions progress with an increase in the number of degrees of freedom (d.o.f.). When the number of d.o.f. reaches six, the end effector can take up any position and posture in the working space. Therefore it is easily understood that six is the number of d.o.f. necessary and sufficient to handle an object in any position and posture [1].

How many active d.o.f. are necessary and sufficient to realize the minimum walking functions in multi-legged locomotion machines that do not need any dynamic balance control? What model exists? A machine in which redundant d.o.f. are removed may enable drastic simplification of control and fast locomotion, both at the expense of some flexibility, i.e. restriction of foot placement, and fixation of gait.

Although consideration of the d.o.f. seem to be fundamental in the development of multi-legged locomotion machines, the precise considerations of the walking functions and d.o.f. have not yet been obtained in previous work.

This paper defines the minimum walking functions and reports some basic considerations on the number of active d.o.f. necessary and sufficient to realize these functions.

2. NUMBER OF DEGREES OF FREEDOM OF LEGGED LOCOMOTION MACHINES

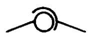
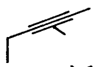
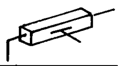
2.1. Number of degrees of freedom of a kinematic pair and number of degrees of freedom of a mechanism

The number of d.o.f. of a kinematic pair signifies the number of freedoms in a joint between two bodies. For instance, if two bodies are rigidly fixed, the number of these freedoms is zero. As constraints are progressively removed one-by-one, the body acquires one, two, three, etc. d.o.f., until it is eventually free with its six freedoms. If the number of constraints is denoted by u, and the number of freedoms by f, then

$$u+f=6. \tag{1}$$

The values of f and u in typical kinematic pairs are given in Table 1.

Table 1.
Examples of kinematic pairs

Item / Pair	Schematic diagram	Number of freedoms	Number of constraints
Spherical pair		3	3
Cylindrical pair		2	4
Turning pair		1	5
Prismatic pair		1	5

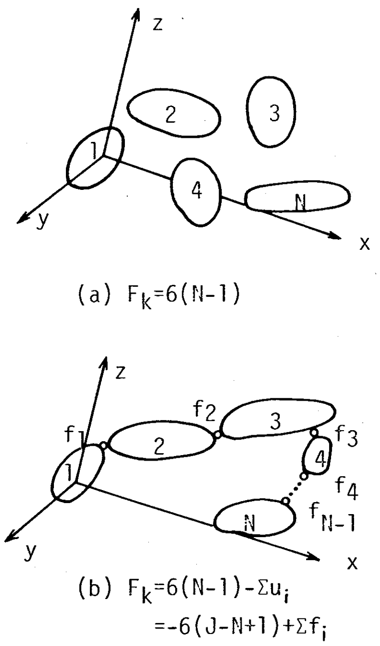


Figure 1. Degrees of freedom of a mechanism.

Let us consider the system of N bodies shown in Fig. 1. Given N bodies all completely unconstrained, then any one of them can be chosen as a reference body, and the total number of relative d.o.f. is $6(N-1)$. Now impose independent constraints between the bodies, namely, joints in the form of profiles contacting one another. The number of degrees of constraint of the *i*th joint is denoted by u_i, and may take any value from 1 to 5. Now, with J working joints between a total of N bodies, the number of relative d.o.f. [2] can be written as equation (2)

$$F_k = 6(N-1) - \Sigma u_i. \tag{2}$$

From equation (1), u_i can be replaced by $6-f_i$, then equation (2) can be expressed in terms of freedoms f_i, which is usually more convenient than degrees of constraint u_i, and

$$F_k = -6(J-N+1) + \Sigma f_i. \tag{3}$$

The relative d.o.f. is equal to the number of independent variables which must be specified in order to locate all the bodies of the mechanism relative to one another. It is also called the mobility or *d.o.f. of a mechanism* [2]. The term in parentheses in equation (3) expresses the number of independent loops which exist in the N body system. Therefore, in a legged machine, the number of d.o.f. of a mechanism is smaller than the total number of freedoms of kinematic pairs. This is because several independent loops exist between the body and the ground. However, in a manipulator in which no loop exists except handling, these two values coincide. Equations (2) and (3) correspond to the equations [3, 4] which Morecki *et al.* used in calculating the d.o.f. of a human hand.

2.2. Classification of degrees of freedom

The d.o.f. of a kinematic pair can be classified into two types according to whether or not actuators are supplied. The d.o.f. with actuators is called the *active d.o.f.* and that without actuators is called the *passive d.o.f.*

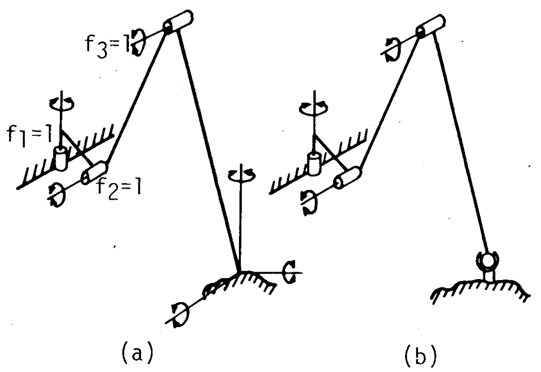


Figure 2. Typical leg mechanism and types of degrees of freedom.

A typical leg configuration [5, 6] is shown in Fig. 2. If f_1, f_2 and f_3 are all active d.o.f. as shown in Fig. 2(a), the foot can select its position freely in the movable space. Generally, the foot has three rotary passive d.o.f. [5] between it and the ground. Although point contact with friction is ideal for this purpose, it is possible to supply the equivalent d.o.f. substantially even though surface contact takes place if the foot and leg are connected through the spherical pair given in Table 1. On the basis of this knowledge, the d.o.f. which we use can generally be classified as shown in Table 2. In a legged locomotion machine with n d.o.f., what kind of d.o.f. does the number stand for? Although an accurate definition has not yet been obtained it is considered that in general n expresses the total number of active d.o.f. This number, in a given machine, cannot change in any phase, and therefore it intuitively expresses the controllability of a machine, while the d.o.f. of a mechanism becomes an index to estimate the flexibility of a machine.

Table 2.
Classification of d.o.f.

```
                                       ┌active D.O.F.
           D.O.F. of a kinematic ┤
          ┌pair                        └passive D.O.F.
 D.O.F. ┤
          └D.O.F. of a mechanism
```

2.3. Degrees of freedom and functions in previous machines

The total number of active d.o.f. in previous machines is listed in Table 3, where the asterisk indicates machines that cannot realize two-dimensional walking without slipping between the foot and the ground. The machines with the largest number of active d.o.f. in Table 3 are those that can select the position of the foot freely and have $3k$ active d.o.f., where k is the number of legs. Since wheeled vehicles, which are most popular for a well-organized terrain, and crawler-type vehicles, which can proceed on small irregular terrains, can move freely using only two active d.o.f., evidently legged machines have a greater number of active d.o.f. With k_l lifted legs, the d.o.f. of a mechanism of machines which have three active d.o.f. per leg is expressed as

$$F_k = 6 + 3k_l \quad (k_l = 0, 1, \ldots k-3). \tag{4}$$

Equation (4) is derived using equation (3) (see Appendix 1). It indicates that the body can move in any direction and rotate around any axis, even when all the legs are on the ground ($k_l = 0$). Under existing conditions, for which there are few practical machines in the world, we wonder whether this function is really necessary for the walking function.

In this paper we define the minimum walking functions from the standpoint of walking and concentrate on the problem of how many active d.o.f. are necessary and sufficient to realize these functions.

3. FUNCTIONAL CLASSIFICATION OF LEGGED LOCOMOTION MACHINES

In order to examine closely the minimum walking functions of legged machines, we first classify the functions according to the following walking levels:

Level 1: One-dimensional walking on a flat plane with static stability.
Level 2: Two-dimensional walking on a flat plane with static stability.
Level 3: Keeping the body height horizontal on an irregular terrain as well as two-dimensional walking and static stability.

Table 3.
Active d.o.f. of legged locomotion machines

4-legs	Tokyo Institute of Technology [7]	12
	Mechanical Engineering Laboratory [8]	8*
6-legs	Ohio State University [5]	18
	Moscow State University [6]	18
	Carnegie-Mellon University [9]	18
	Odetics. Inc. [10]	18
	Paris University [11]	12*
	Roma University [12]	12*
8-legs	Komatsu Ltd. [13]	10

Level 4: Keeping the body height constant as far as the leg length can allow in addition to the function of level 3.

Level 5: Selecting foot placement freely within its movable space.

To maintain static stability, the legged machine must satisfy the following conditions [14]:

(i) There must be more than three support legs.

(ii) The projection of the centre of gravity must always lie within the support polygon formed by the support legs.

The main reason for keeping the body horizontal in level 3 is to maintain an adequate stability margin [15], defined for the legged machine as the minimum distance between the centre of gravity and the support line during a cycle period. Other important reasons include simplification of control, improvement of energetic efficiency based on the consideration of a gravitationally decoupled actuating system [16] and improvement of payload.

Keeping the absolute body height constant in level 4 is required from the viewpoint of energetic efficiency. It is desirable to keep the absolute body height constant in practical actuators without energy-storing systems to save energy. The energy consumed depends only on the relative difference in height for an actuator with an energy-storing system, even though the body accompanies the severe up-and-down movements.

Two methods are considered in Fig. 3. One keeps the body height constant by using a slide actuator [Fig. 3(a)]. When the machine goes down a slope, the slide actuator consumes energy corresponding to the potential energy m_0gZ, where m_0 is the mass of the body and Z is the relative height difference. The other method [Fig. 3(b)] is the gravitationally decoupled actuator method proposed by Hirose [16]. Type B is assumed in this study because with this type it is possible to attain no-energy consumption during horizontal movement of the body. The walking difference between levels 3 and 4 is shown in Fig. 4.

On the other hand, although it is possible for the machine with level 4 to select a suitable gait, for instance, a mixed gait with the rotational mode of the body about any axis and a straight walking mode, we consider that such functions are related to

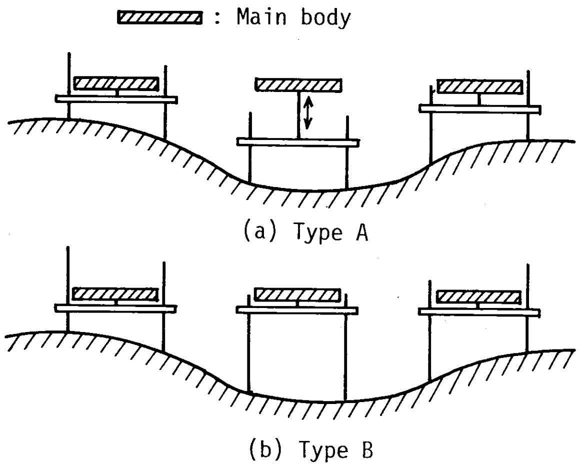


Figure 3. Two methods to keep the body height constant.

the flexibility of locomotion and are not absolutely necessary for a machine with legs. We have thus defined level 4 as the minimum walking function in this study.

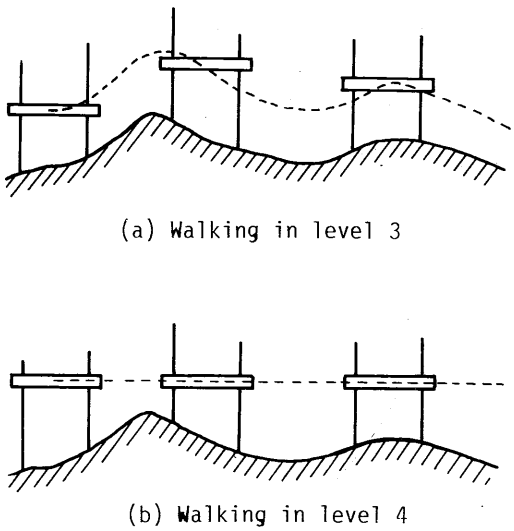


Figure 4. Difference between levels 3 and 4.

4. CONSIDERATIONS ON MINIMUM NUMBER OF ACTIVE DEGREES OF FREEDOM

It is easy to understand that walking in levels 1 and 2 can be realized by one and two active d.o.f. [17], respectively, if cams and links are suitably connected, and surely such numbers are necessary and sufficient.

In order to extract a model with the minimum number of active d.o.f., let us classify the legged machine into two types:

(i) a machine that realizes locomotion and static stability by using only the leg's freedoms (insect type);

(ii) a machine that realizes locomotion and static stability by using different freedoms (separated type).

The insect type imitates an insect or animal and almost all of the earlier machines [5–7, 9, 10] belong to this type. The separated type exists only in the artificial machine. With very few examples [18], this type of machine is designed without any consideration of balance, and if the machine does not satisfy the static stability condition for a regulated walking pattern, the active d.o.f. for changing the centre of gravity is added. Since the shift of the centre of gravity is limited within the horizontal plane and the aim is to set it in the support polygon, only one active d.o.f. is adequate, if at all necessary. Since the separated type has the potential for becoming a legged model with fewer active d.o.f. than the insect type, we focused on this type in the following discussion.

4.1. Considerations on necessity

Let us consider a four-legged model which has the minimum number of legs capable of maintaining static stability (because it is expected that fewer legs lead to a reduction in the number of active d.o.f.). Fig. 5 shows a generalized four-legged model, where $\mathbf{L}_i$ and $\mathbf{F}_i$ $(i=1, 2, \ldots, 4)$ are respectively vectors expressing the hip joint and foot joint, respectively; P_G is the projection point of the centre of gravity on the terrain surface; **G** is the vector expressing the position of P_G; **H** is the vector expressing the height between P_G and the body; **n** is the unit vector, its direction being perpendicular to the support triangle; and **i**, **j**, **k** are unit vectors expressing the x, y, z directions, respectively. However, the vector **n** cannot be defined when all the legs are on the ground. Since this is not an essential problem for obtaining the minimum number of active d.o.f., we apply the same **n** determined in a three-legged support phase, even when all the legs are on the ground.

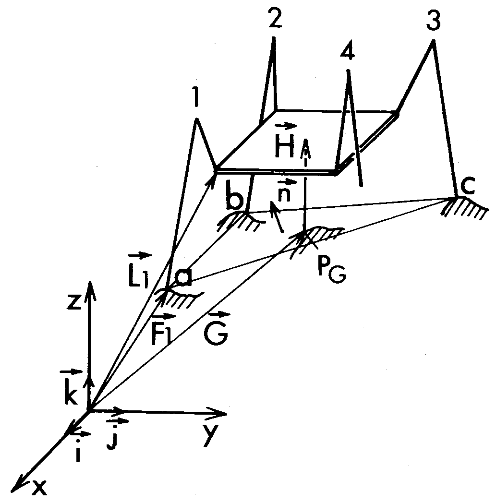


Figure 5. Vector notation of each position of a four-legged model.

The functions of level 4 consist of four parts: (a) two-dimensional walking; (b) keeping the body horizontal; (c) keeping the body height constant; and (d) maintaining static stability. Functions (a). (b) and (c) can be explained using the above-defined vector as follows:

(a) **G** can be determined in any position on the terrain.

(b) $(\mathbf{L}_i - \mathbf{L}_{i+1}) \cdot \mathbf{k} = 0 \quad i = 1, 2.$ (5)

where $\mathbf{A} \cdot \mathbf{B}$ expresses the scalar product of the vectors **A** and **B**.

(c) $Z_{max} = \max\ (\mathbf{L}_i - \mathbf{F}_i) \cdot \mathbf{k}$ (6)

$Z_{min} = \min\ (\mathbf{L}_i - \mathbf{F}_i) \cdot \mathbf{k}$ (7)

where i is the number of the supported leg.

(i) $Z_{max} < h_{max}$ and $Z_{min} > h_{min}$

$$\frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{L}_i \cdot \mathbf{k}) = 0. \tag{8}$$

(ii) $z_{max} = h_{max}$ or $Z_{min} = h_{min}$

It is necessary to change the body height where h_{max} and h_{min} are the maximum and minimum heights capable of moving between the body and foot, respectively.

Function (b) implies that **n** can be turned in any direction, independent of the body posture, and function (c) implies that **H** can be determined in any position within the leg's movable space. Therefore, to realize functions (a), (b) and (c), it is necessary that at least **G**, **n** and **H** be determined freely. The components of **G**, **n** and **H** are expressed by

$$\mathbf{G} = (X_G, Y_G, Z_G) \tag{9}$$

$$\mathbf{n} = (\cos \alpha, \cos \beta, \cos \gamma) \tag{10}$$

$$\mathbf{H} = (0, 0, h) \tag{11}$$

where α, β and γ are the angles between each axis fixed on the body and **n**, respectively, and consequently the following relation exists:

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1. \tag{12}$$

Since **G** expresses the position on the terrain, once the terrain is given Z_G depends on the parameters X_G and Y_G and therefore it can be given by

$$Z_G = Z_G(X_G, Y_G). \tag{13}$$

From equations (9)–(13), the actual number of independent parameters reduces to 5. Since each independent parameter corresponds to the active d.o.f., six (one active d.o.f. for balance is added) is the minimum number to realize the walking in level 4.

4.2. Considerations on the sufficiency

Let us consider the four-legged model illustrated in Figs 6(a) and 7 to show the sufficiency of the necessary condition. This model is equipped with four legs (four active d.o.f.) and has a body (one active d.o.f.) capable of sliding, a weight capable of rotating (one active d.o.f.), and one passive d.o.f. in the connecting point between its front leg unit (or rear leg unit) and the body, where the spherical pair is assumed to be between the foot and the ground. Therefore this model has six active and two passive d.o.f. The basic sequence of locomotion shown in Fig. 6(b) is realized by sliding the

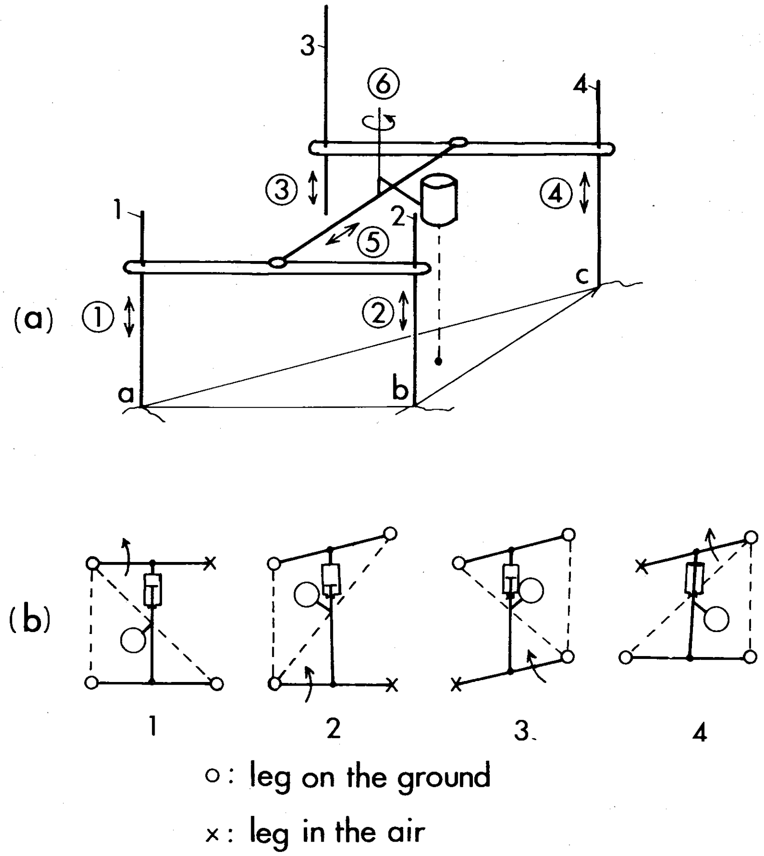


Figure 6. Proposed four-legged model with six d.o.f. and the walking sequence.

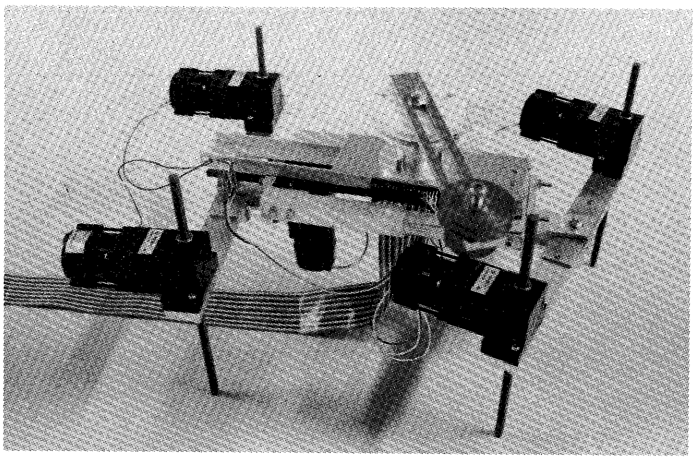


Figure 7. Overall view of the proposed four-legged model.

body after putting the centre of gravity into the next support triangle and lifting the prearranged leg. The relation between the d.o.f. of the mechanism and the active d.o.f. is shown in Table 4 (the leg's number and the number of active d.o.f. are given in Fig. 6). The body must be lifted and lowered by actuating three legs synchronously as shown in Table 4, because independent operation of the legs causes the foot to slip on the ground, as shown in Fig. 8. Therefore when one leg is in the air, the d.o.f. of the mechanism reduces to four, even though the total number of active d.o.f. is still six (see Appendix 1). One d.o.f. of the mechanism corresponds to the up-and-down movement of the body, two d.o.f. correspond to the sliding movement of the body and the up-and-down movement of the lifted leg, and the final one corresponds to the shift of the centre of gravity. However, a relation between the d.o.f. of the mechanism and the body's attitude does not essentially exist in the system. Although with the up-and-down movement of the body this d.o.f. should be realized by actuating three legs on the ground, this d.o.f. is lost according to the assumption that there is no slipping between the foot and the ground. Although the function of body attitude is lost in the concept of d.o.f. of the mechanism, functionally it is not lost altogether. When a lifted leg is positioned according to the terrain and a new support triangle is determined, the resultant attitude of the support triangle changes in relation to the body. As this movement cannot be realized with walking, we cannot include it in the concept of the d.o.f. of the mechanism.

Table 4.
Relation between active d.o.f. and d.o.f. of a mechanism

lifted leg's number	item of D.O.F. of a mechanism			
	up-and-down movement of main body	up-and-down movement of lifted leg	sliding movement of main body	shift of the center of gravity
1	② ③ ④	①	⑤	⑥
2	① ③ ④	②	⑤	⑥
3	① ② ④	③	⑤	⑥
4	① ② ③	④	⑤	⑥
all legs are on the ground	①②③④	×	×	⑥

○: active D.O.F.

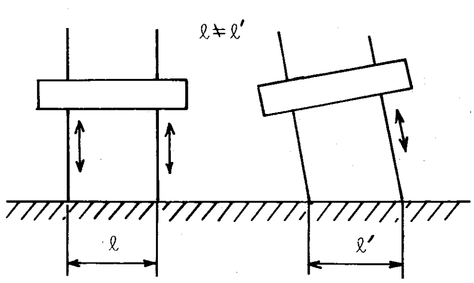


Figure 8. Slipping between foot and ground in the proposed four-legged model. (This is caused by independent operation of each leg on the ground.)

On the basis of these considerations, since it is assumed that the proposed four-legged model can accomplish functions (b), (c) and (d), the problem of whether or not this model can realize the walking in level 4 leaves us with only having to examine the possibility of (a), that is, two-dimensional walking. This point is demonstrated in Figs 9 and 10. The movement of L_1L_2 from the initial to the final state can be understood as the combination of the rotation of L_1L_2 and the movement of P_1. L_1L_2 can be easily rotated, and results in the problem of how to move point P_1 from P_1 (x_s, y_s) to $P_1(x_f, y_f)$, because the same idea can also be applied to point P_2. Now let point L_1 rotate around point L_2 with $\Delta\theta$ (positive for clockwise motion) and then let point L_2 rotate around point L_1' with $-\Delta\theta$. $P_1(x, y)$ is transferred to $P_1(x+\Delta x, y+\Delta y)$ by these two movements, as shown in Fig. 10(a). Next let point L_1' rotate around point L_2' with $-\Delta\theta$, and let point L_2' rotate around point L_1'' with $\Delta\theta$ as shown in Fig. 10(b).

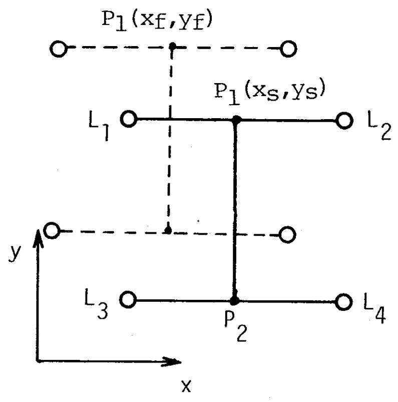
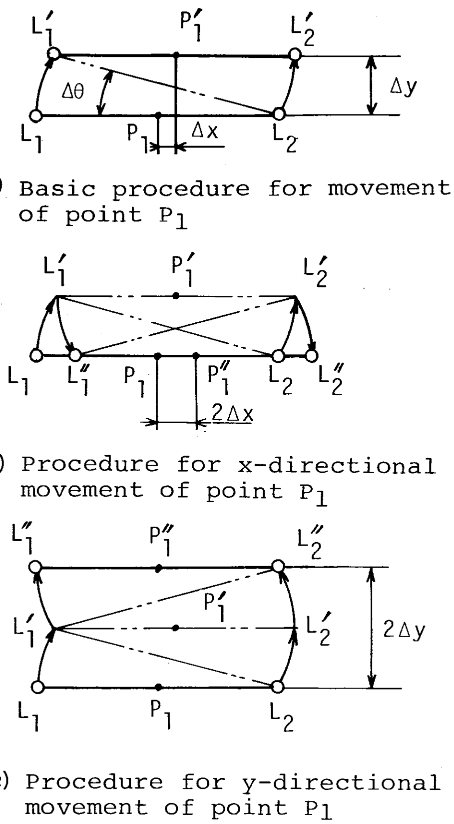


Figure 9. Two-dimensional expression of the proposed model.

Finally, $P_1(x, y)$ is transferred to $P_1(x+2\Delta x, y)$ and eventually a series of these movements [(a) and (b)] corresponds to movement in the x-direction by $2\Delta x$. Furthermore, let point L_2' rotate around point L_1' with $-\Delta\theta$, and let point L_1' rotate around point L_2'' with $\Delta\theta$, as shown in Fig. 10(c). $P_1(x, y)$ is then transferred to $P_1(x, y+2\Delta y)$ and eventually a series of these movements [(a) and (c)] corresponds to movement in the y-direction by $2\Delta y$. Therefore, these explanations show that point P_1 can be moved independently for x and y, and that point P_1 can be moved in any position by mixing two-dimensional movements. Since the same idea can also be applied to the rotation and movement of L_1L_2, it was proved that this model satisfies function (a). Six active d.o.f. in this model correspond to the number discussed in Section 4.1. Consequently, the proposed four-legged model can realize the walking in level 4 with a minimum number of active d.o.f. It is also possible to change the body direction freely if a suitable control algorithm is applied to this model.

5. CONCLUSION

Classification of d.o.f. was carried out and it was shown that the d.o.f. of a mechanism and the total number of active d.o.f. become an important index for considering the d.o.f. of a multi-legged locomotion machine.



(a) Basic procedure for movement of point P_1

(b) Procedure for x-directional movement of point P_1

(c) Procedure for y-directional movement of point P_1

Figure 10. Walking procedure to reach any position in the x–y plane.

We classified into several levels functionally multi-legged locomotion machines that do not need any dynamic balance control.

The necessary and sufficient conditions for active d.o.f. were examined by defining these minimum walking functions for a legged locomotion machine capable of proceeding on an irregular terrain. As a result, it was revealed that six is the number of active d.o.f. necessary and sufficient to realize the minimum walking functions.

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APPENDIX 1

As an example, let us consider a four-legged machine with three active d.o.f. As shown in Figs 11 and 12, only two phases can be considered.

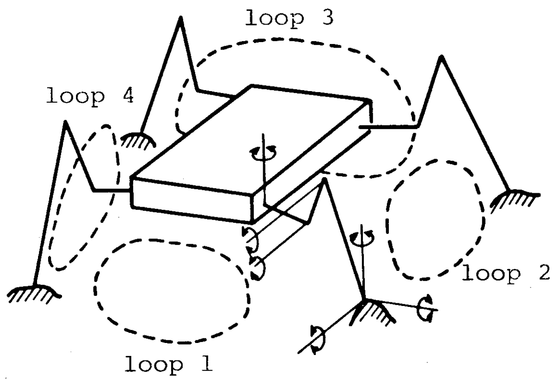


Figure 11. Four-legged machine with three active d.o.f. for each leg. (All legs are on the ground.)

(i) All legs are on the ground

Although several loops can be generated in this particular phase, the number of independent loops is actually only three. On the other hand, since each leg has three active d.o.f. and three passive d.o.f., the total number of d.o.f. of a kinematic pair is six for each leg. Eventually the d.o.f. of a mechanism [equation (3)] becomes $F_k = -6\times 3 + 4\times 6 = 6$.

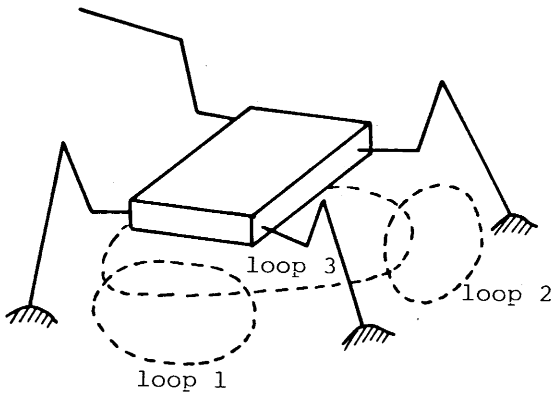


Figure 12. Four-legged machine with three active d.o.f. for each leg. (Three legs are on the ground.)

(ii) Three legs are on the ground

The number of independent loops reduces by two. Since one leg changes from stance to transfer, three passive d.o.f. between the foot and the ground are lost for one leg. Thus, the d.o.f. of the mechanism becomes $F_k = -6\times 2 + 3\times 6 + 3 = 9$.

Now let us assume a k-legged machine with k_1 lifted legs. It is easily proved that the number of independent loops becomes $(k - k_1 - 1)$. Since the total number of active d.o.f. during stance and transfer is equal to $6(k - k_1)$ and $3k_1$, respectively, equation (4) is proved in the following way:

$$\begin{aligned} F_k &= -6(k - k_1 - 1) + 6(k - k_1) + 3k_1 \\ &= 6 + 3k_1 \end{aligned} \tag{A1}$$

where $k_1 = 0, 1, \ldots, k-3$.

APPENDIX 2

Since slipping between the foot and ground is not allowed, it is impossible for the proposed four-legged model to change body attitude from the horizontal plane. Eventually, (i) the movement in the horizontal plane and (ii) the movement in the vertical plane are perfectly decoupled and the proposed model reduces to a combination of two planar mechanisms. The d.o.f. of the mechanism in a planar mechanism is given by

$$F_k = -3(J - N + 1) + \Sigma f_i. \tag{A2}$$

(i) D.o.f. of the mechanism in the horizontal plane

According to Fig. 13(a), $J = 5$, $N = 5$, $f_1 = f_2 = f_3 = f_4 = f_5 = 1$ and therefore $F_k = 2$. These freedoms correspond to sliding movement of the body and rotational movement of the balancing weight.

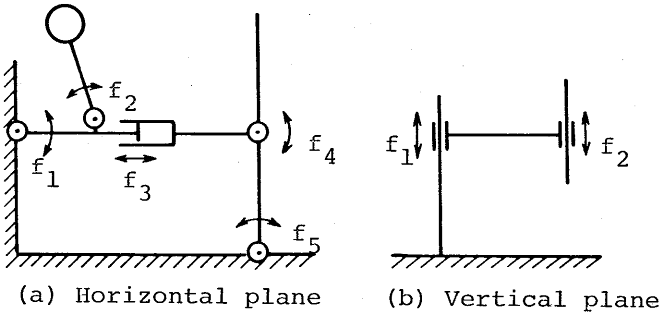
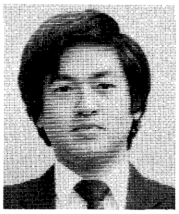


Figure 13. Planar mechanisms of the proposed four-legged model.

(ii) D.o.f. of the mechanism in the vertical plane
According to Fig. 13(b), $J=2$, $N=3$, $f_1=f_2=1$ and therefore $F_k=2$. These d.o.f. correspond to the up-and-down movement of the body and the lifted leg.

ABOUT THE AUTHORS



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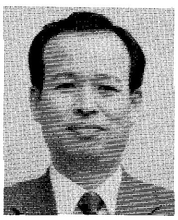

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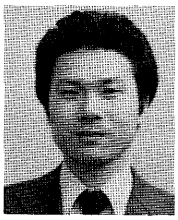

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